Establishing and Maintaining the Parametric Resonance Condition

Compensation for aberrations in a solenoidal cooling channel

Chromatic aberration
Angular aberration
Magnetic field nonlinearity

Introduction

For parametric resonances in a solenoidal channel

- Canonical angular momentum $\hat{M}$ is conserved.
- The ½ integer parametric resonance is radial and driven by small modulations of the solenoidal field strength
- At resonance, the transverse momentum grows and the minimum radial position decreases:

$$\rho_{\min} \times p_{\perp} \rightarrow const = \hat{M}$$

- The challenge is to have all particles reach the minimum radial position at the same place in the channel (at the absorber positions).
- However, the tune spread due to chromatic and other aberrations prevents this and it is necessary to compensate these aberrations.

Expansion of Aberrations

- The local focusing strength can be expressed in terms of the field and velocity:

$$\frac{1}{f^2} \propto \left[\frac{B(z,\rho)}{2\gamma\upsilon_z}\right]^2 \equiv \frac{1}{4}B^2\left(\frac{1}{\upsilon_z{}^2} - 1 - \frac{\upsilon_\perp{}^2}{\upsilon_z{}^2}\right) \qquad (1)$$

- The first term on the right contains only the tune spread due to chromaticity;
  - this can be proved as follows:
  - In an RF field (needed for energy loss compensation) particles experience synchrotron oscillations. The average value of the longitudinal velocity is determined by the RF field; this value has no spread, i.e.:
- $$\frac{1}{\upsilon_z} = \frac{1}{\upsilon_0} + \Delta\frac{1}{\upsilon_z}; \quad <\Delta\frac{1}{\upsilon_z}> = 0$$
  - $\Delta\upsilon_z{}^{-1}$ is due to the oscillation of particle momentum: , $\Delta\upsilon_z{}^{-1} \approx -\tilde{p}/\gamma^2\upsilon p$
  - Then to first order in $\tilde{p}$:
- $$\frac{1}{\upsilon_z{}^2} \approx \frac{1}{\upsilon_0{}^2} - \frac{2}{(\gamma\upsilon)^2}\frac{\tilde{p}}{p}$$

Chromatic Aberration Compensation

- Requiring fast synchrotron oscillations, where

$$k_s \gg \Lambda_c \,,$$

implies that, on average, the betatron wave-length becomes constant.

The impact of chromatic aberrations is then reduced to a small spread of betatron phase. This contributes to an increase of the beam size at the absorber:

$$\sigma^2 = \frac{\overline{\vartheta^2}}{k_s^2}\frac{\overline{\tilde{p}^2}}{p^2}$$

Kinematic or Angular Aberration

- The last term of equation (1) , proportional to the square of the pitch angle of the particle motion, represents a contribution to the tune spread due to the transverse momentum spread. Since the term is negative it can only decrease the focusing strength.
  - (It is interesting that this result is opposite to the conclusion that would be reached by observing the effect of focusing at fixed energy. The explanation of this “renormalization” is that, by synchronizing the longitudinal motion of particles via synchrotron oscillations, the RF field creates a transverse amplitude - energy correlation in the bunched beam.)

Compensation of Angular Aberration

- The angular aberration, which leads to the decrease of transverse tune with amplitude, can be compensated by using the fringe fields of an alternating solenoid channel to generate a tune increase with amplitude.

  - Generally, one has to satisfy two conditions in order to compensate for non-linear terms – since the non-linear tune is a function of the two action variables, I and $\hat{M}$. In terms of the reduced (conservative) Hamiltonian function, there are three second order terms in this function: I^2, $\hat{M}^2$, and $\alpha\hat{M}I$, where α is a constant axial vector (associated with the solenoid field). However, for a symmetrically alternating solenoid such a physical vector does not exist, so $\alpha = 0$. Here, one has to satisfy only one condition: to compensate for the term $\propto I^2$ in the Hamiltonian function. Such an optimum point can be found by simulation accompanied by analytical guidance.

Compensation of Angular Aberration (Continued)

The condition for compensation is:

$$2Alk^2 = 1$$

where *l* and *A* are the length and aperture of a single solenoid, and $k = 2\pi / \lambda$